



TITLE:

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CITATION:

Kawai, Shin ...[et al]. Macroeconomic Implications of Coordination Failure in a Multisector Cournot-Nash Model(Mathematical Economics). 数理解析研究所講究録 2006, 1488: 202-215

ISSUE DATE:

2006-05

URL:

<http://hdl.handle.net/2433/58176>

RIGHT:

# Macroeconomic Implications of Coordination Failure in a Multisector Cournot-Nash Model

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## Abstract

It is known that the multisector Cournot-Nash models may exhibit multiple, Pareto-ranked equilibria. The main condition is that the price elasticity of demand is sufficiently low at the survival level of consumption. This paper investigates the macroeconomic implications of the model by analyzing the intra-industrial interaction and the circulative structure of multisector economy. It is shown that the subjective aggregate supply curve is downward sloping as well as the objective demand curve. It is also found that there is strategic complementarity across firms within each industry as well as across sectors. Furthermore, there exhibits not only a form of strategic complementarity but also that of strategic substitutability across sectors.

JEL classifications: E1, D43, D58, L16,

Keywords: Coordination Failure, Strategic Complementarity, AD-AS approach.

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# 1 Introduction

It is known that the multisector Cournot-Nash models may exhibit multiple, Pareto-ranked equilibria. That is, a coordination failure may exist (e.g., Bryant 1983; Cooper and John 1988). Among others, the condition is that the price elasticity of demand is sufficiently low at the survival level of consumption (Heller 1986). This paper investigates the macroeconomic implications of the Heller model by analyzing the intra-industrial interaction and the circulative structure of multisector economy.

There are two main trends of macroeconomic theory based on microfoundation. One is the New Classical (e.g., Lucas 1972, later the RBC literature). According to Hahn and Solow (1995),

It proposes that the actual economy can be read as if it is acting out or approximating the infinite-time discounted utility maximizing program of a single, immortal representative agent. ... There is simply no possibility of coordination failure. ...

Another trend is the New Keynesian. Although there are several types of models, the common factor is that they construct the models that deviate from the Walrasian mechanism. According to Mankiw and Romer (1991).

There are two questions that one may ask about any theory of economic fluctuations.

- Does the theory violate the classical dichotomy? ...
- Does the theory assume that real market imperfections in the economy are crucial for understanding economic fluctuations? ...

New Keynesian economics answers an emphatic yes to both of these questions.

In this paper, we would answer “no” for the first question, and “yes” for the second one. As in Heller (1986), the money doesn’t appear in the model. The real market imperfection that we consider is the imperfect competition.

Following Ch.4 in Cooper (1999), there are two types of imperfect competitions on which macroeconomic models based. One is well-known the model of monopolistic competition represented by Blanchard and Kiyotaki (1987).

Another type of the model is the multisector Cournot-Nash models represented by Hart (1982), Heller (1986)<sup>1</sup>. The economy consists of multiple sectors. In each sector, firms behave strategically as an oligopolist taking as given the position of the industry demand curve, determined by the level of activity in other sectors, and the output level of other firms in their own sector. Cooper (1999) says that there is usually strategic substitutability across sellers, and the economy exhibits a form of strategic complementarity across sectors. However, the model we will present here, originated by Heller (1986), has completely different characteristics from Cooper and the other oligopoly theorists in mind.

The purpose of this paper, hence, is to present the rich implications of the intra- and inter-industrial interactions, and to analyze it as an AD-AS approach. It is found that there is a strategic complementarity across firms within each industry, and that there exhibits both forms of strategic complementarity and strategic substitutability across sectors. It is shown that the curve of subjective aggregate supply is downward sloping in addition to that of the objective demand<sup>2</sup>. The subjective aggregate supply curve is derived by the loci of the partial Cournot-Nash equilibria.

In section 2, we present the Heller model and analyze the intra- and inter-industrial relations. In section 3, we analyse the objective demand and subjective supply curve like AD-AS approach. In section 4, we conclude it.

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<sup>1</sup>We would like to notice that the seminal model of this type can be found in Nikaido (1975), since it is not mentioned in Cooper (1999).

<sup>2</sup>Nikaido (1975) analyses the model of Leontief system of two goods and two sectors. It is said that the objective demand curves need not be downward sloping. Our model is a partial in the sense of Hart (1982). That is, we restrict the type of utility function through the analysis.

## 2 The Heller Model

In this section, we would construct the modified version of Heller model. Consider the structure of modern economy. The economy consists of multiple sectors. Consumers spent their income to their favorite goods. It commonly happen that those goods are not coincide with the goods of industry they work for. Suppose that the worker doesn't directly consume their working industry's products. Another character of modern economy is that the technological progress is highly developed. Because of that, the workers need different skills for new jobs at different industry. Thus, we assume that the labors don't change the industry. For the sake of simplicity, we restrict the number of sectors as two. There are  $n$  households and  $m$  firms in each sector. In each sector, firms produce the homogeneous final consumption goods.

### 2.1 Consumer

Suppose that consumer preferences are defined by Stone-Geary type with  $n$  goods, that is,

$$u(x_1, x_2, \dots, x_n) = \prod_{i=1}^n (x_i - \underline{x}_i)^{\beta_i}, \quad \beta_i > 0. \quad (1)$$

This is also called linear expenditure system (LES), since the derived demand functions become linear with respect to real income. Assume that  $n = 2$ , and that one is consumption good  $c$ , another is leisure  $l$ . Taking logarithms we get

$$u(c, l) = \beta_1 \log(c - \underline{c}) + \beta_2 \log l. \quad (2)$$

Notice that we assume  $\underline{l} = 0$ .

Suppose that type 1 representative consumer solves the following problem.

$$\begin{aligned} \max_{c_2, l_1} \quad & \beta_1 \log(c_2 - \underline{c}) + \beta_2 \log l_1, \\ \text{s.t.} \quad & p_2 c_2 \leq w_1 L_1^s + r_1, \\ & l_1 = \bar{L} - L_1^s, \end{aligned} \quad (3)$$

where  $c_2, l_1$  are consumption of sector 2 and leisure of type 1 respectively. The profits of good 1 firms are equally distributed to the type 1 households.

It is denoted by  $r_1 \equiv \frac{1}{n} \sum_{i=1}^m \pi_1^i$ . There is the survival level of consumption  $\underline{c} (> 0)$ .  $\bar{L}$  and  $L_1^s$  denote the initial endowment of labor and the labor supply of type 1 household respectively.

Similarly, the problem of type 2 households has the symmetric structure. That is,

$$\begin{aligned} \max_{c_1, l_2} \quad & \beta_1 \log(c_1 - \underline{c}) + \beta_2 \log l_2, \\ \text{s.t.} \quad & p_1 c_1 \leq w_2 L_2^s + r_2, \\ & l_2 = \bar{L} - L_2^s. \end{aligned} \tag{4}$$

Notice that  $\beta_1$  and  $\beta_2$  are common across types. For the sake of brevity, we consider the problem of type 1.

The first order condition is

$$\frac{\beta_1 l_1}{\beta_2(c_2 - \underline{c})} = \frac{p_2}{w_1}. \tag{5}$$

The demand function of type 1 becomes

$$c_2(p_2, w_1, r_1) = \frac{\beta_2 \underline{c}}{\beta} + \frac{\beta_1(w_1 \bar{L} + r_1)}{\beta p_2}, \tag{6}$$

where  $\beta = \beta_1 + \beta_2$ .

For simplicity, suppose that  $n = 1$ . Then, the inverse demand function of market 2 can be written as

$$p_2(c_2, w_1, r_1) = \frac{\beta_1(w_1 \bar{L} + r_1)}{\beta c_2 - \beta_2 \underline{c}}. \tag{7}$$

The inverse of price elasticity of demand becomes

$$\frac{Ep_2(c_2, w_1, r_1)}{Ec_2} = \frac{\beta c_2}{\beta c_2 - \beta_2 \underline{c}}. \tag{8}$$

Similarly, the above equations for type 2 can be derived by changing the number.

## 2.2 Firm

There are  $m$  firms within a sector. For simplicity, suppose that the production function is identical in each sector and  $m = 2$ . Then, for sector  $j$

( $j = 1, 2$ ), the production function is given by

$$y_j = \alpha L_j^d, \quad \alpha > 0. \quad (9)$$

Consider the firm  $i$  of sector 1 ( $i = 1, 2$ ). She maximizes her profits taking as given the subjective inverse demand function (7) and the outputs of the other firms within the sector. Then, the profit function becomes

$$\pi_1(y_1^i, Y_1', w_2, r_2) = \left[ p_1(y_1^i, Y_1', w_2, r_2) - \frac{w_1}{\alpha} \right] y_1^i \quad \text{for } i = 1, 2. \quad (10)$$

The first order condition of firm  $i$  of sector 1 is

$$p_1 \left( 1 - \frac{y_1^i}{Y_1} \cdot \frac{Ep_1}{Ec_1} \right) = \frac{w_1}{\alpha} \quad \text{for } i = 1, 2. \quad (11)$$

From (11), we can derive the response functions for firm 1, 2, in sector 1 whose shapes appear in Figure 1. From the above discussions, we can establish the following proposition.

**PROPOSITION 1.** *There are two partial symmetric Cournot-Nash equilibria in each industry. They exhibit strategic complement at each equilibrium.*

**proof .** *Combining (11), (7) and (8), it can be written as*

$$\frac{\beta_1(w_2\bar{L} + r_2)}{\beta Y_1 - \beta_2\bar{c}} \left( 1 - \frac{\beta y_1^1}{\beta Y_1 - \beta_2\bar{c}} \right) = \frac{w_1}{\alpha}. \quad (12)$$

*Taking the implicit derivative of (12) with respect to  $y_1^1$  and  $y_1^2$ , we obtain*

$$\frac{dy_1^1}{dy_1^2} = \frac{\beta(Y_1 - 2y_1^1) - \beta_2\bar{c}}{-2(\beta(Y_1 - y_1^1) - \beta_2\bar{c})}. \quad (13)$$

*Substituting  $y_1^1 = y_1^2 = y_1^*$  into (13),*

$$\frac{dy_1^1}{dy_1^2} = \frac{\beta_2\bar{c}}{2(\beta y_1^* - \beta_2\bar{c})} > 0. \quad (14)$$

*This implies that there exhibits a strategic complement at the equilibria. The proof of existence of two C-N equilibria is shown in the proof of Proposition 4.*

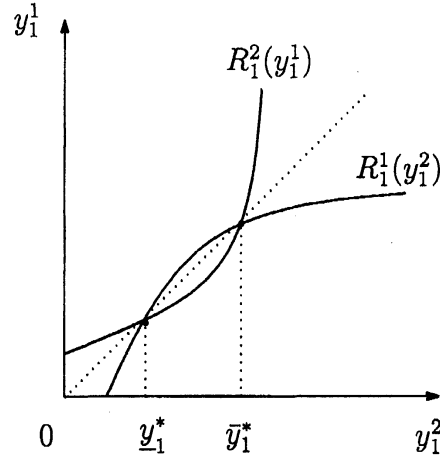


Figure 1: Reaction Function of Intra-Industry

### 2.3 Feasibility Condition

Consider the feasibility conditions. Since it is symmetric in intra-industry, the equilibrium output of each firm would be equal. That is  $y_1^1 = y_1^2 = \dots = y_1^*$ . Because of identical utility function, demands for consumption of consumers in each type are equal. Hence, the feasibility condition for good 1 market is  $nc_1 = my_1$ .

On the other hand, the feasibility condition for type 1 labor market can be written as  $nL_1^s = mL_1^d$ .

From the production function and the time constraint of Labor supply,  $L_1^s = \bar{L} - l_1$ , the feasibility condition can be derived as

$$l_1 = \bar{L} - \frac{c_1}{\alpha}. \quad (15)$$

### 2.4 General Cournot-Nash Equilibrium

Consider the symmetric Nash equilibrium (SNE) in each sector. Suppose that  $w = w_1 = w_2 = 1$ . From consumer's f.o.c. (5),

$$p_2(c_2, l_1) = \frac{\beta_1 l_1}{\beta_2(c_2 - \underline{c})} \quad (16)$$



Substituting the feasibility condition (15) into (16)

$$p_2(c_2, c_1) = \frac{\beta_1 \left( \bar{L} - \frac{c_1}{\alpha} \right)}{\beta_2(c_2 - \underline{c})}. \quad (17)$$

Since we focus on the symmetric Nash equilibria (SNE) in each sector, the equilibrium output is equal;  $y_1^i = y_1 = Y_1/m$  and  $y_2^i = y_2 = Y_2/m$ . From (11) and  $w_2 = 1$ , firm's f.o.c. in good 2 industry can be written as:

$$\frac{Ep_2}{Ec_2} = m \left[ 1 - \frac{1}{p_2\alpha} \right]. \quad (18)$$

Substituting (17) into RHS in (18), we obtain,

$$\frac{Ep_2}{Ec_2} = m \left[ 1 - \frac{\beta_2(c_2 - \underline{c})}{\beta_1(\alpha\bar{L} - c_1)} \right]. \quad (19)$$

Substituting (8) into LHS in (19),

$$\frac{\beta c_2}{\beta c_2 - \beta_2 \underline{c}} = m \left[ 1 - \frac{\beta_2(c_2 - \underline{c})}{\beta_1(\alpha\bar{L} - c_1)} \right]. \quad (20)$$

Similarly, since the structure of industry is symmetric, the corresponding condition of (19) for market 1 must hold simultaneously at the general Cournot-Nash equilibrium.

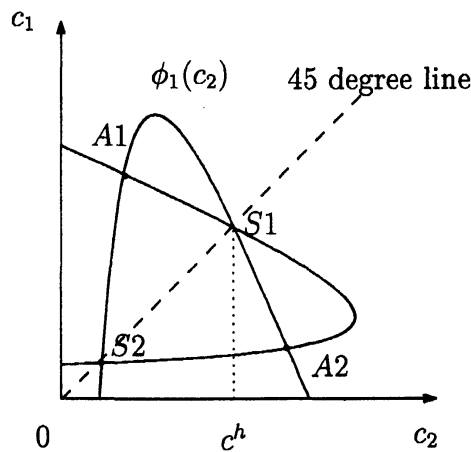


Figure 2: General Cournot-Nash Equilibria

**PROPOSITION 2.** *Suppose that the following three conditions (21), (22) and (23) are satisfied, then there are four general Cournot-Nash equilibria (GCNE) in this economy. Two of those are symmetric, the other two are asymmetric. The symmetric GCNE are Pareto-ranked.*

**proof .** *From Theorem 4 in Heller (1986), The set of corresponding conditions for the symmetric multiple Pareto-ranked equilibria is that there is  $c' < c^*$ , such that*

$$p(c') \left[ 1 - \frac{Ep(c', r)}{mEc} \right] > \frac{1}{\alpha}, \quad (21)$$

and

$$\lim_{c \rightarrow \underline{c}} \frac{Ep(c, r)}{Ec} = \frac{\beta}{\beta_1} > m. \quad (22)$$

*For asymmetric GCNE. Solving the equation (20) for  $c_1$ . Then, we obtain the function;  $c_1 = \phi_1(c_2)$ . From Figure 2, if the above two conditions are satisfied, then there are two intersection on the 45 degree line. Let  $c^h$  be the higher solution of the two. Then, the sufficient condition of existence of asymmetric GCNE is*

$$\frac{d\phi_1(c^h)}{dc_2} < -1. \quad (23)$$

### 3 AD-AS Analysis

In this section, we consider the macroeconomic implication of multisector Cournot-Nash (C-N) model.

#### 3.1 Objective Demand Curve

From the utility maximization problem (4), the labor supply function is derived to

$$L_1^s(w_1, p_2, r_1) = \frac{\beta_1 \bar{L}}{\beta} + \frac{\beta_2(p_2 \underline{c} - r_1)}{\beta w_1}. \quad (24)$$

Suppose that an aggregate labor supply is full-employed by the firms. Notice that we don't consider firm's profit maximization for a moment. Then, the share distribution of each type 1 consumer becomes the following:

$$\begin{aligned} r_1 &= \frac{m}{n}\pi = \frac{m}{n}(p_1 y_1 - w_1 L_1^d), \\ &= \frac{m}{n}(\alpha p_1 - w_1)L_1^d, \quad (y_1 = \alpha L_1^d) \\ &= (\alpha p_1 - w_1)L_1^s(w_1, p_2, r_1). \quad (L_1^d = \frac{n}{m}L_1^s(w_1, p_2, r_1)) \end{aligned} \quad (25)$$

Substituting (25) into (24) and solving for  $L_1^s$ , we obtain,

$$L_1^s(w_1, p_2, p_1) = \frac{\beta_1 w_1 \bar{L} + \beta_2 p_2 \underline{c}}{\beta_1 w_1 + \beta_2 \alpha p_1}. \quad (26)$$

We can call it as the objective labor supply function of type 1 consumer. Using (26), the share distribution can be written as

$$r_1 = (\alpha p_1 - w_1) \frac{\beta_1 w_1 \bar{L} + \beta_2 p_2 \underline{c}}{\beta_1 w_1 + \beta_2 \alpha p_1}. \quad (27)$$

Substituting (27) into the demand function (6), and aggregate the demand for good 2,

$$C_2(p_2, p_1, w_1) = n \left( \frac{\beta_2 \underline{c}}{\beta} + \frac{\beta_1 w_1 \bar{L}}{\beta p_2} + \frac{\beta_1 (\alpha p_1 - w_1) (\beta_1 w_1 \bar{L} + \beta_2 p_2 \underline{c})}{\beta p_2 (\beta_1 w_1 + \beta_2 \alpha p_1)} \right). \quad (28)$$

Since we focus on the symmetric equilibria and the aggregate demand function is homothetic of degree 0, let  $p_1 = p_2 = p$  and  $w_1 = w_2 = 1$ ,

$$C(p) = n \left( \frac{\beta_2 \underline{c}}{\beta} + \frac{\beta_1 \bar{L}}{\beta p} + \frac{\beta_1 (\alpha p - 1) (\beta_1 \bar{L} + \beta_2 p \underline{c})}{\beta p (\beta_1 + \beta_2 \alpha p)} \right). \quad (29)$$

This is the objective demand function.

**PROPOSITION 3.** *Suppose that  $\alpha \bar{L} > \underline{c}$ , then the objective demand curve is downward sloping.*

**proof .** *The proof is straight forward. Taking derivative of (29) with respect to  $p$ ,*

$$\frac{dC(p)}{dp} = - \frac{n\alpha\beta_1\beta_2(\alpha\bar{L} - \underline{c})}{(\beta_1 + \beta_2\alpha p)^2} \quad (30)$$

Hence if  $\alpha \bar{L} > \underline{c}$ , then

$$\frac{dC(p)}{dp} < 0. \quad (31)$$

The assumption  $\alpha\bar{L} > \underline{c}$  is quite plausible because it implies that outputs produced by all initial endowments exceed the survival level of consumption.

### 3.2 Subjective C-N Curve

We suppose that firms don't know the objective demand curve. Hence firms face subjective demand. Thus, from (7), the inverse demand function can be written as

$$p_2(Y_2, I_1) = \frac{\beta_1 n I_1}{\beta Y_2 - \beta_2 n \underline{c}}, \quad (32)$$

where  $I_I \equiv w_1 \bar{L} + r_1$ , and  $Y_2$  denotes the total output of industry 2.

$$\frac{Ep(Y_2, I_1)}{EY_2} = \frac{\beta Y_2}{\beta Y_2 - \beta_2 n \underline{c}}. \quad (33)$$

From (11), (7), and (33), the f.o.c. of the representative firm 2 becomes

$$\frac{n\beta_1 I_1}{\beta Y_2 - \beta_2 n \underline{c}} \left( 1 - \frac{y_2^i}{Y_2} \cdot \frac{\beta Y_2}{\beta Y_2 - \beta_2 n \underline{c}} \right) = \frac{w_2}{\alpha}. \quad (34)$$

Solving (34) for  $y_2^i$  taking as given  $Y_2$ , and setting  $w_1 = w_2 = 1, n = 1$  for simplicity, we obtain

$$y_2^i = \left( 1 - \frac{(\beta Y_2 - \beta_2 \underline{c})}{\alpha \beta_1 I_1} \right) \frac{\beta Y_2 - \beta_2 \underline{c}}{\beta}. \quad (35)$$

Aggregating (35) through the sector, and solving for  $Y_2$ , then we can obtain the following function

$$Y_2(I_1) = \frac{\beta_2 \underline{c}}{\beta} + \frac{(m-1)\alpha\beta_1 I_1 \pm \sqrt{((m-1)\alpha\beta_1 I_1)^2 - 4m\beta_2 \underline{c}(\alpha\beta_1 I_1)}}{2m\beta}. \quad (36)$$

**PROPOSITION 4.** *Subjective C-N curve is downward sloping.*

**proof .** Let  $D = ((m-1)\alpha\beta_1 I_1)^2 - 4m\beta_2 \underline{c}(\alpha\beta_1 I_1)$ . If  $D \geq 0$ , that is,

$$I_1 \geq \frac{4m\beta_2 \underline{c}}{(m-1)^2 \beta_1 \alpha} \equiv \underline{I}, \quad (37)$$

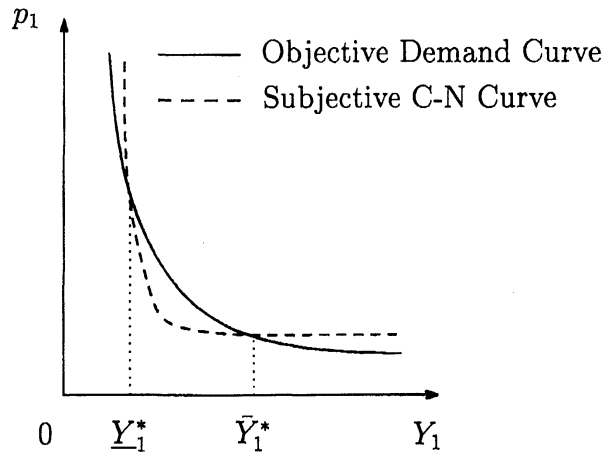


Figure 3: AD-AS Analysis

then there exists solution for  $Y_2$ . Moreover, if  $D > 0$ , then there are two solutions. Denote  $Y_2^h(I_1)$  and  $Y_2^l(I_1)$  as higher outputs and lower outputs, respectively. From (36), for all  $I_1 > \underline{I}$ ,

$$\frac{dY_2^h(I_1)}{dI_1} > 0, \quad \frac{dY_2^l(I_1)}{dI_1} < 0. \quad (38)$$

From the subjective demand function (32), we can obtain the corresponding price levels to  $Y_2(I_1)$ .

$$p_2(I_1) = \frac{\beta_1 I_1}{\beta Y_2(I_1) - \beta_2 \underline{c}}. \quad (39)$$

Then, the sets of prices and outputs  $(p_2(I_1), Y_2(I_1))$  creates the subjective C-N curve appears in Figure 3.

## 4 Conclusion

We have shown the fruitful results of the multiple Cournot-Nash model. We would like to note that the assumptions are not so much different from Walrasian. The differences are the multisector structure and the imperfect competition at the goods markets. These assumptions seem to suit the developed countries well. In developed countries, there are lots of industries and the

relationships of those industries are more intricate. It seems odd to suppose that there is a powerful auctioneer who can adjust the entire markets simultaneously. If there is not such an auctioneer in the economy, and the prices are determined by market by market, then the state of economy could easily fluctuate among these equilibria. Furthermore, there is the possibility that the economy would not reach any equilibria and fluctuates forever. We can say that this is another type of coordination failure. In order to avoid these situations, some authorities, such as the government and central bank, should announce the direction of that they are proceeding to. Another effective policy is to make the market more competitive by the means of allowing the new entry to the industry. This latter policy is aimed to change the structure of the economy itself.

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